TOPIC 1

The amount of sand in the lower bulb of an hourglass is directly proportional to the time since the glass was turned over.

Quantities and Relationships

TOPIC 1 OVERVIEW

Quantities and Relationships

How are the key concepts of *Quantities and Relationships* organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin with an introductory lesson on the problem-solving model. They will use this model throughout the course when solving problems. They then analyze real-world scenarios. These scenarios move beyond the linear relationships familiar from Grade 7 and Grade 8 to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs' shape and structure and then sort them according to defined characteristics.

Students are introduced to the definitions of *function*, *domain*, and *range*. Building on their knowledge from previous grades, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

Math Representation

A function family is a group of functions that share certain characteristics.

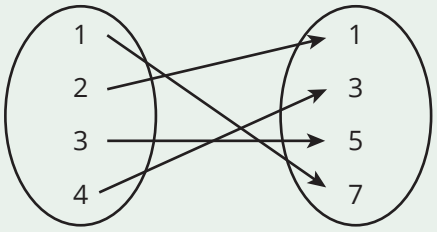
- The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.
- The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers and b is greater than 0 but not equal to 1.
- The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a is not equal to 0.

What is the entry point for students?

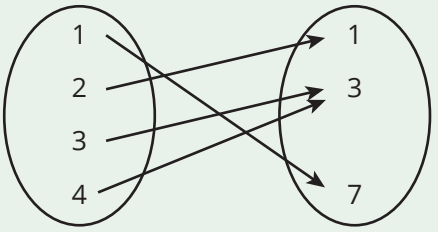
Students have defined independent and dependent variables and have used them to write equations and create tables for various relationships. They have defined a function and used linear functions to model the relationship between two quantities.

Math Representation

In each mapping shown, the domain is {1, 2, 3, 4}.



The range is {1, 3, 5, 7}.



The range is {1, 3, 7}.

Each mapping represents a function because no input, or domain value, is mapped to more than one output, or range value.

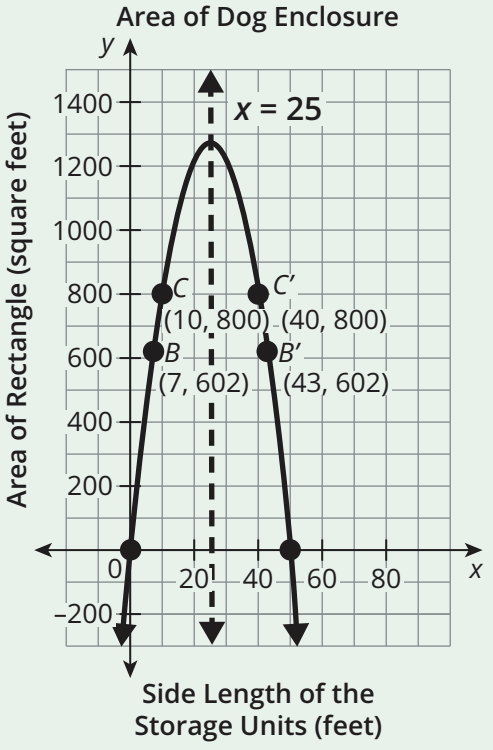
Why is *Quantities and Relationships* important?

Recognizing patterns and structure in multiple function representations allows students to generalize patterns across function families.

Math Representation

The axis of symmetry is the vertical line that passes through the vertex and divides the parabola into two mirror images.

By analyzing the symmetric point, you can see that the *y*-coordinates of symmetric points are the same and the horizontal distance of each symmetric point from the axis of symmetry is the same. In this situation, the axis of symmetry of $x = 25$ indicates that a side length of 25 feet for the storage area will yield the maximum area for the dog pen.



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Quantities and Relationships* when they can:

- Choose appropriate scale and origin for graphs.
- Identify the appropriate unit of measure for each variable or quantity.
- Analyze a graph and state the key characteristics of the graph.
- Use a problem situation to explain what the key features of a graph mean in real-world context.
- Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.
- Recognize a linear, exponential, or quadratic function by its equation or graph.
- Evaluate functions, expressed in function notation, given one or more elements in their domain.
- Determine the domain and range and the independent and dependent quantities in a relationship.

How do the activities in *Quantities and Relationships* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout *Quantities and Relationships*, applying mathematics to everyday life (A.1A), using the problem-solving model (A.1B), communicating mathematical ideas through multiple representations (A1.D), and connecting mathematical ideas (A.1F) are highlighted. Students search for patterns in tables, equations, and scenarios. They examine the structure of these function representations to identify common characteristics of function types. They should notice that the equations of graphs in the same family all take the same general form.

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Strategically encourage students to keep a bilingual math journal, recording reflections and background knowledge on new topics, in either written or verbal format, with added visuals for clarity. Incorporate journal excerpts into a shared word wall or digital bilingual glossary, with a focus on highlighting cognates.

NEW KEY TERMS

- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

NEW SYMBOL

Symbol	Description
$f(x)$	Function notation

MODULE 1, TOPIC 1 PACING GUIDE

165-Day Pacing

1 Searching for Patterns

TOPIC 1: Quantities and Relationships

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.B, 1.C, 1.E, 1.F, 2.C, 2.E, 2.I, 3.D, 3.E, 3.H, 4.C, 4.E, 4.G, 4.H, 5.B, 5.F

Topic Pacing: 14 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
	Introduction to the Problem-Solving Model and Learning Resources	Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving process. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process, not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Model Questions to Ask, and TEKS mathematical process standards which are located in the Course Guide. These materials should always be available to students throughout the course. **Materials Needed:** (located in the Course Guide) Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Model Questions to Ask, TEKS Mathematical Process Standards	**A.3C**	1
1	Understanding Quantities and Their Relationship	Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario. **Materials Needed:** Glue Sticks, Scissors	**A.3C** **A.7A** **A.9D**	2
2	Analyzing and Sorting Graphs	Students begin this lesson by cutting out 13 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow. **Materials Needed:** Scissors, Graph Cards (located at the end of the lesson)	**A.3C** **A.7A** **A.9D**	1

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Recognizing Functions and Function Families**	The definitions *function* and *function notation* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function forms to their graphs and then identify the function family to which they belong. The terms *increasing function*, *decreasing function*, and *constant function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing, decreasing, and constant intervals. The terms *function family*, *linear function*, and *exponential function* are then defined, and students sort the increasing constant and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined as well as the term *quadratic function*. Students sort the functions with an absolute minimum or absolute maximum. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs. **Materials Needed:** Graphs from *Analyzing and Sorting Graphs*, Graphing Technology, Glue Sticks	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	3
4	**Recognizing Functions by Characteristics**	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph. **Materials Needed:** Graphs from *Analyzing and Sorting Graphs*, Problem-Solving Model Graphic Organizer	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				4

*Bold TEKS = Readiness Standard

MODULE 1, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.3C**	TEKS: **A.3C, A.7A, A.9D**			TEKS: **A.3C, A.7A, A.9D**
Introduction to the Problem-Solving Model and Learning Resources **GETTING STARTED** **ACTIVITY 1** **TALK THE TALK**	**LESSON 1** Understanding Quantities and Their Relationships **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**LESSON 2** Analyzing and Sorting Graphs **GETTING STARTED** **ACTIVITY 1** **TALK THE TALK**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
	TEKS: **A.2A, A.3C, A.6A, A.7A,** A.9A, **A.9D,** A.12A			
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**LESSON 3** Recognizing Functions and Function Families **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 3** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11	**Day 12**	**Day 13**	**Day 14**	
TEKS: **A.2A, A.3C, A.6A, A.7A,** A.9A, **A.9D,** A.12A				
LESSON 4 Recognizing Functions by Characteristics **GETTING STARTED** **ACTIVITY 1**	**LESSON 4** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**	

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are four Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice set of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.